Fast Computation of 3-D Eddy Current Problems with Integral Formulations using Wavelet Approximation Patterns

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***Abstract*—Large transient 3-D eddy current problems with integral formulations require a compression technique for the fully populated system matrix. The Hierarchical Block Wavelet Compression, which is independent of the number of degrees of freedom and suitable for parallelization, will be used to reduce the memory costs to linear dependencies. The computation cost for the assembly of the system matrix is quadratic as every integral value has to be computed. This effort is reduced by so called Wavelet Approximation Patterns, which allows for computing only absolutely necessary integral values. The efficiency of the Wavelet Approximation Patterns applied to eddy current problems will be analyzed.**

I. Introduction

Computing three-dimensional (3-D) eddy current problems with the volume integral equation method (VIEM) results in a fully populated system matrix (SM) with a memory requirement of $O(N^2)$ for a number of N unknowns. This can be reduced to $O(N)$ by using compression techniques like the Block Wavelet Compression (BWC) [1] or the Hierarchical Block Wavelet Compression (HWC) [2] for an arbitrary number of unknowns N.

The computational costs can be reduced in two cases. In the first case the effort can be decreased while assembling the system matrix. Due to sorting the numbering of the degrees of freedom (DoF), the SM establishes patterns at each block (Fig. 1). Therefore, only a few integral values have to be computed if the blocks can be well approximated by patterns. This is done by so called Wavelet Approximation Patterns (WAP), which were successfully applied to 3-D radiative heat transfer BEM problems [3]. The WAP performs the transform and compression at once. In the second case the computation time of the matrix vector product (MVO) is reduced if calculating in wavelet space with a compressed matrix.

The WAP reduces the memory requirement as well as the computation time, which becomes significant for large-scale problems.

II. Eddy Current Formulation

Eddy currents in a non-magnetic, electrically conductive region V_c can be described by the following equation [4]:

$$\frac{1}{\kappa}\boldsymbol{J}(\boldsymbol{r}) + \frac{\mu_0}{4\pi}\frac{\partial}{\partial t}\int_{V_c}\frac{\boldsymbol{J}(\boldsymbol{r'})}{|\boldsymbol{r}-\boldsymbol{r'}|}dV' = -\frac{\partial}{\partial t}\boldsymbol{A}_s(\boldsymbol{r'}). \quad (1)$$

$\boldsymbol{J}(\boldsymbol{r'})$ is the eddy current density, κ the electrical conductivity, μ_0 the magnetic field constant, and $\boldsymbol{A_s}(\boldsymbol{r'})$ the magnetic vector potential of a time-varying source.

Using suitable shape functions $\boldsymbol{W}_i$ and the Galerkin method, the eddy current problem is described by the linear system [4][5]

$$\{U\} = [R]\{I\} + [L]\frac{d\{I\}}{dt}, \quad (2)$$

with

$$R_{ij} = \frac{1}{\kappa}\int_{V_c}\boldsymbol{W}_i(\boldsymbol{r})\cdot\boldsymbol{W}_j(\boldsymbol{r})dV, \quad (3)$$

$$L_{ij} = \frac{\mu_0}{4\pi}\int_{V_c}\int_{V_c}\frac{\boldsymbol{W}_i(\boldsymbol{r})\cdot\boldsymbol{W}_j(\boldsymbol{r})}{|\boldsymbol{r}-\boldsymbol{r'}|}dV'dV, \quad (4)$$

$$U_i = -\frac{\partial}{\partial t}\int_{V_c}\boldsymbol{A}_s(\boldsymbol{r'})\cdot\boldsymbol{W}_i(\boldsymbol{r})dV. \quad (5)$$

The eddy currents can be calculated by using the electric vector potential $\boldsymbol{T}$ for the current density $\boldsymbol{J} = \nabla\times\boldsymbol{T}$. In this case $\boldsymbol{W}_i = \nabla\times\boldsymbol{N}_i$, where $\boldsymbol{N}_i$ are edge-element-based shape functions of $\boldsymbol{T}$ [4]. In the second case $\boldsymbol{W}_i = \boldsymbol{N}_i$, with $\boldsymbol{N}_i$ being edge-element-based shape functions of $\boldsymbol{J}$ [6]. The gauge assuring the uniqueness of the solution is in both cases realized by a tree-cotree decomposition of the edges [4][6].

III. BWC and HWC

To allow for an arbitrary number of degrees of freedom (#DoF), a completely new wavelet compression technique was developed called BWC [1] and later extended to the HWC [2] by involving the geometrical aspects of the model.

A. Block Wavelet Compression

For the BWC, the system matrix is split into k blocks of size $2^p \times 2^q$, where $(p, q) \in \mathbb{W}_0$. Both side lengths of each blocks can be chosen in common to the *Ancient Egyptian Multiplication*, which displays any positive number by a sum of 2^p related numbers. Afterwards, each block is transformed by a 2-D FWT of matching p and q steps and thresholded in wavelet space $V_{p,q}$ to thin out the coefficients [1]. The physical compression is done by switching each – now – sparse block to a *compressed row storage*. To solve the BEM problem over the various number of k blocks at compressed state, the right hand side (RHS) and the solution vector (SV) are also split accordingly to the horizontal or vertical block structure. Therefore, each part of the RHS is transformed by a 1-D FWT of p steps [1]. The BEM problem can be solved by using an iterative solver like GMRES for all compressed sub spaces $V_{p,q}$ if an orthonormal wavelet is chosen, e.g. orthonormal Haar wavelet $W_{Haar} = \frac{1}{\sqrt{2}}\{1, -1\}$.

B. Hierarchical Block Wavelet Compression

The HWC extends the BWC by taking the geometry into account. Hereby, the geometrical model is fragmented by *kd-tree*, which groups the numerical elements and allows for fast calculation of distances between the groups. Hence, the indices i, j covered by a block are mapped back into geometry and by the kd-tree shown how large the distance between source and destination is. Small distances indicate important integral results due to a decay of the integral kernel for eddy current problems of $\frac{1}{dist}$ and the block is refined by hierarchical matrices [2]. For large distances the block is kept in size of $2^p \times 2^q$. The gain of the HWC is to predict the importance of areas in the system matrix, which allows for an adjustable compression rate. The iterative solving is performed in hierarchical wavelet sub spaces in common to an extended scheme of the BWC [2].

The final step in both compression techniques is to reverse transform the solution of the problem into the normal space. BWC and HWC reduce the memory costs from quadratic to linear dependency. The computational cost of the transform can be neglected compared to the computation efforts of the integrals [1][2].

IV. Wavelet Approximation Patterns

Using the BWC or HWC reduces only the quadratic growth of memory but not the computation costs. Therefore, a new compression technique based on the BWC and HWC is developed, which reduces the memory and computational costs.

Sorting the degrees of freedom results in a structured system matrix (Fig. 1).

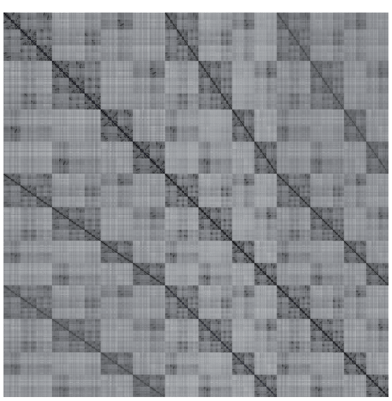

Fig. 1. Discretized visualization of the system matrix

Fig. 1 shows the integral values of the SM of an eddy current problem using the VIEM with edge-element-based shape functions. High coefficient values are represented by dark points. Different structure types like toeplitz or stairs (Fig. 2) appear locally.

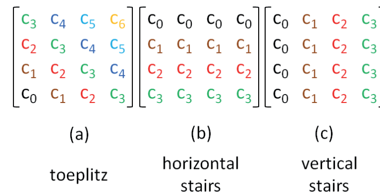

Fig. 2. Different structure types in matrix blocks

In Fig. 2 the three most occurring pattern types are shown. To control the error caused by applying a wrong pattern to the block, a matching factor m is calculated for predefined patterns with e. g. toeplitz or stair structure. By setting a minimum matching factor, the user can control the accuracy of the approximation. Other parameters like compression, threshold, and block size can be set by the user.

The main steps of the WAP algorithm are:

1) Divide system matrix into blocks.
2) Calculate some integral values of the block.
3) Calculate match factor of the block.
4) If resulting match factor is larger then user limit, set remaining entries, else calculate remaining entries.
5) Wavelet transform block-by-block.
6) Compression by thresholding.
7) Solve SLE in wavelet space block-by-block.
8) Reverse transform of found solution.

The WAP shall be analyzed for 3-D eddy current problems for the two possible methods described in Sec. II. The error characteristics will be presented for different number of unknowns, block sizes, compression rates, and match factors.

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